

# Quasinormal Modes of Self-Dual Warped $\text{AdS}_3$ Black Hole in Topological Massive Gravity

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**ABSTRACT:** We consider the various perturbations of self-dual warped  $\text{AdS}_3$  black hole and obtain the exact expressions of quasinormal modes by imposing the vanishing Dirichlet boundary condition at asymptotic infinity. It is expected that the quasinormal modes agree with the poles of retarded Green's functions of the dual CFT. Our results provide a quantitative test of the warped AdS/CFT correspondence.

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## 1. Introduction

Topological massive gravity (TMG) is described by the theory of three dimensional Einstein gravity with a gravitational Chern-Simons correction and the cosmological constant [1, 2]. The well-known spacelike warped  $\text{AdS}_3$  black hole [3] (previously obtained in [4]), which is a vacuum solution of topological massive gravity, is conjectured to be dual to a two dimensional conformal field theory (CFT) with non-zero left and right central charges [5].

Recently, a new class of solutions in three dimensional topological massive gravity named as self-dual warped  $\text{AdS}_3$  black hole is proposed by Chen et al in [6]. The metric is given by

$$ds^2 = \frac{1}{\nu^2 + 3} \left[ -(x - x_+) (x - x_-) d\tau^2 + \frac{1}{(x - x_+) (x - x_-)} dx^2 + \frac{4\nu^2}{\nu^2 + 3} \left( \alpha d\theta + \frac{1}{2} (2x - x_+ - x_-) d\tau \right)^2 \right], \quad (1.1)$$

where  $x_+$  and  $x_-$  are the location of the outer and inner horizons respectively, and we have set  $l = 1$  for simplicity.

The self-dual warped  $\text{AdS}_3$  black hole, which is asymptotic to the warped  $\text{AdS}_3$  spacetime, is locally equivalent to spacelike warped  $\text{AdS}_3$  spacetime. This solution is of the isometry group  $\text{U}(1) \times \text{SL}(2, \mathbb{R})$ . It is shown in [6] that, under the consistent boundary condition, the  $\text{U}(1)$  isometry is enhanced to a Virasoro algebra with nonvanishing left central charge, while the  $\text{SL}(2, \mathbb{R})$  isometry becomes trivial with the vanishing right central charge,

$$c_L = \frac{4\nu}{\nu^2 + 3}, \quad c_R = 0. \quad (1.2)$$

It is conjectured that the self-dual warped  $\text{AdS}_3$  black hole is dual to a two dimensional chiral CFT, which suggests a novel example of warped  $\text{AdS}/\text{CFT}$  dual.

The left and right temperatures of CFT can not be read directly from the coordinates transformation of locally identification to warped  $AdS_3$  space. They can be defined with respect to the Frolov-Thorne vacuum [7]. Considering the quantum field with eigenmodes of the asymptotic energy  $\omega$  and angular momentum  $k$ , and assuming that the left and right charges  $n_L, n_R$  are  $k$  and  $\omega$ , the corresponding Boltzmann factor in terms of these variables is given by  $e^{-\frac{\omega-k\Omega}{T_H}} = e^{-\frac{n_L}{T_L} - \frac{n_R}{T_R}}$ , where the left and right temperatures are defined by

$$T_L = \frac{\alpha}{2\pi}, \quad T_R = \frac{x_+ - x_-}{4\pi}. \quad (1.3)$$

In this paper, we want to investigate another interesting aspect of the self-dual warped  $AdS_3$  black hole inspired by the  $AdS/CFT$  corresponding. It is expected that the quasinormal modes exactly agree with the location of the poles of retarded Green's functions of the dual CFT. In an remarkable paper [8] (see also [9]), the quantitative agreement is confirmed by the analytical calculations of quasinormal modes of various perturbations for BTZ black hole. More recently, it is shown that the warped  $AdS_3$  black hole preserves the same property as BTZ black hole [10, 11]. One can refer to [12, 13, 14, 15] for numerical investigations about this aspect. We have studied the various perturbations of self-dual warped  $AdS_3$  black hole and found that the wave equations can be exactly solved by the hypergeometric function. This observation allows us to analytically calculate the quasinormal modes for various perturbations. Comparing with the previously results reported in [10], we show that the quasinormal modes are just of the forms predicted by dual CFT. The results may provide a quantitative test of the warped  $AdS/CFT$  correspondence.

This paper is organized as follows. In the following three sections, we will calculate the quasinormal modes of scalar, vector and spinor perturbations respectively and compare them with the prediction of warped  $AdS/CFT$  dual. The last section is devoted to conclusion and discussion.

## 2. Quasinormal modes of scalar field perturbation

In this section, we will calculate the quasinormal modes of scalar field perturbation in the background of self-dual warped  $AdS_3$  black hole. Let us consider the scalar field  $\Phi$  with the mass  $m$  in the background of self-dual warped  $AdS_3$  black hole, where the wave equation is given by the Klein-Gordon equation

$$\left( \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) - m^2 \right) \Phi = 0. \quad (2.1)$$

Because the self-dual warped  $AdS_3$  black hole possesses two killing vectors  $\partial_\tau$  and  $\partial_\theta$ , the scalar field wave function  $\Phi(\tau, x, \theta)$  can be expanded in eigenmodes as following

$$\Phi(\tau, x, \theta) = e^{-i\omega\tau + ik\theta} R(x), \quad (2.2)$$

where  $\omega$  and  $k$  are the energy and angular momentum of scalar field, respectively. Substituting this expression for scalar field perturbation into the Klein-Gordon equation, one

can get the radial wave equation

$$\left[ \partial_x ((x - x_+)(x - x_-)\partial_x) + \frac{\left(\omega + \frac{x_+ - x_-}{2\alpha}k\right)^2}{(x - x_+)(x_+ - x_-)} - \frac{\left(\omega - \frac{x_+ - x_-}{2\alpha}k\right)^2}{(x - x_-)(x_+ - x_-)} \right] R(x) + \left( \frac{3(\nu^2 - 1)}{4\nu^2} \frac{k^2}{\alpha^2} - \frac{1}{\nu^2 + 3} m^2 \right) R(x) = 0. \quad (2.3)$$

This equation can be analytically solved by the hypergeometric function. By changing the variable to

$$z = \frac{x - x_+}{x - x_-}, \quad (2.4)$$

the radial wave equation can be rewritten in the form of hypergeometric equation

$$z(1 - z) \frac{d^2 R(z)}{dz^2} + (1 - z) \frac{dR(z)}{dz} + \left( \frac{A_s}{z} + B_s + \frac{C_s}{1 - z} \right) R(z) = 0, \quad (2.5)$$

where the parameters  $A_s$ ,  $B_s$  and  $C_s$  are given by

$$\begin{aligned} A_s &= \left( \frac{k}{2\alpha} + \frac{\omega}{x_+ - x_-} \right)^2, \\ B_s &= - \left( \frac{k}{2\alpha} - \frac{\omega}{x_+ - x_-} \right)^2, \\ C_s &= \frac{3(\nu^2 - 1)}{4\nu^2} \frac{k^2}{\alpha^2} - \frac{1}{\nu^2 + 3} m^2. \end{aligned} \quad (2.6)$$

According to the definition, the quasinormal modes of black hole must be purely ingoing at the horizon. So we are just interested in the solution with the ingoing boundary condition at the horizon. The solution of radial wave equation with the ingoing boundary condition is explicitly given by the hypergeometric function

$$R(z) = z^{\alpha_s} (1 - z)^{\beta_s} F(a_s, b_s, c_s, z), \quad (2.7)$$

where

$$\alpha_s = -i\sqrt{A_s}, \quad \beta_s = \frac{1}{2} - \sqrt{\frac{1}{4} - C_s}, \quad (2.8)$$

and

$$\begin{aligned} c_s &= 2\alpha_s + 1, \\ a_s &= \alpha_s + \beta_s + i\sqrt{-B_s}, \\ b_s &= \alpha_s + \beta_s - i\sqrt{-B_s}. \end{aligned} \quad (2.9)$$

Using the following transformation relation of hypergeometric function [16]

$$\begin{aligned} F(a, b, c; z) &= \frac{\Gamma(c)\Gamma(c - a - b)}{\Gamma(c - a)\Gamma(c - b)} F(a, b, a + b - c + 1; 1 - z) \\ &+ (1 - z)^{c - a - b} \frac{\Gamma(c)\Gamma(a + b - c)}{\Gamma(a)\Gamma(b)} F(c - a, c - b, c - a - b + 1; 1 - z), \end{aligned} \quad (2.10)$$

one can find the leading asymptotic behaviour ( $z \rightarrow 1$ ) of the solution

$$R(z) \simeq z^{\alpha_s} (1-z)^{\beta_s} \frac{\Gamma(c_s)\Gamma(c_s - a_s - b_s)}{\Gamma(c_s - a_s)\Gamma(c_s - b_s)} . \quad (2.11)$$

Next, in order to find the quasinormal modes, one has to impose the boundary condition at asymptotic infinity. The condition that the flux vanishes at asymptotic infinity is just a perfect one. In this paper, we will use the equivalent Dirichlet condition that the field is vanishing at asymptotic infinity. By imposing the vanishing Dirichlet boundary condition at infinity, one can find the following relation

$$c_s - a_s = -n , \quad \text{or} \quad c_s - b_s = -n , \quad (2.12)$$

which give the quasinormal modes of scalar perturbation

$$\begin{aligned} k &= -i(2\pi T_L)(n + h_L) , \\ \omega &= -i(2\pi T_R)(n + h_R) , \end{aligned} \quad (2.13)$$

with the left and right conformal weights of the operator dual to scalar fields

$$h_L^s = h_R^s = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{3(1-\nu^2)}{4\nu^2} \frac{k^2}{\alpha^2} + \frac{1}{\nu^2 + 3} m^2} . \quad (2.14)$$

One can see that these modes coincide with the poles in the retarded Green's function obtained in [6]. So, for the scalar perturbation, our calculation indicates that the quasinormal modes of self-dual warped black hole are exactly predicted by the dual CFT.

### 3. Quasinormal modes of vector field perturbation

In this section, we calculate the quasinormal modes of vector perturbation. We consider the massive vector field described by the first order differential equation

$$\epsilon_\lambda^{\alpha\beta} \partial_\alpha A_\beta = -m A_\lambda . \quad (3.1)$$

Assuming that the vector field takes the form

$$A_\mu = e^{-i\omega\tau + ik\theta} \phi_\mu(x) , \quad (3.2)$$

one can derive the equations of motion

$$-m\phi_x = \frac{g_{\tau\tau}}{\sqrt{-g}} (ik\phi_\tau + i\omega\phi_\theta) , \quad (3.3)$$

$$\frac{d\phi_\tau}{dx} = \frac{g_{xx}}{\sqrt{-g}} \left[ \left( -\frac{\omega k}{m} + mg_{\tau\theta} \right) \phi_\tau - \left( \frac{\omega^2}{m} + mg_{\tau\tau} \right) \phi_\theta \right] , \quad (3.4)$$

$$\frac{d\phi_\theta}{dx} = \frac{g_{xx}}{\sqrt{-g}} \left[ \left( \frac{k^2}{m} + mg_{\theta\theta} \right) \phi_\tau - \left( -\frac{\omega k}{m} + mg_{\tau\theta} \right) \phi_\theta \right] . \quad (3.5)$$

From Eq.(3.4) and (3.5), one can get the following wave equation for  $\phi_\theta$  after changing the variables to  $z$

$$z(1-z)\frac{d^2\phi_\theta}{dz^2} + (1-z)\frac{d\phi_\theta}{dz} + \left(\frac{A_\nu}{z} + B_\nu + \frac{C_\nu}{1-z}\right)\phi_\theta = 0, \quad (3.6)$$

with

$$\begin{aligned} A_\nu &= \left(\frac{k}{2\alpha} + \frac{\omega}{x_+ - x_-}\right)^2, \\ B_\nu &= -\left(\frac{k}{2\alpha} - \frac{\omega}{x_+ - x_-}\right)^2, \\ C_\nu &= \frac{3(\nu^2 - 1)}{4\nu^2} \frac{k^2}{\alpha^2} - \frac{m^2 - 2m\nu}{\nu^2 + 3}. \end{aligned} \quad (3.7)$$

The solution with the ingoing boundary condition at the horizon is given by

$$\phi_\theta = z^{\alpha_\nu} (1-z)^{\beta_\nu+1} F(a_\nu + 1, b_\nu + 1, c_\nu, z), \quad (3.8)$$

where

$$\alpha_\nu = -i\sqrt{A_\nu}, \quad \beta_\nu = -\frac{1}{2} + \sqrt{\frac{1}{4} - C_\nu}, \quad (3.9)$$

and

$$\begin{aligned} c_\nu &= 2\alpha_\nu + 1, \\ a_\nu &= \alpha_\nu + \beta_\nu + i\sqrt{-B_\nu}, \\ b_\nu &= \alpha_\nu + \beta_\nu - i\sqrt{-B_\nu}. \end{aligned} \quad (3.10)$$

From Eq.(3.5), one can find

$$\phi_\tau = \bar{A}_\nu \phi_\theta + \bar{B}_\nu \frac{\phi_\theta}{1-z} + \bar{C}_\nu z \frac{d\phi_\theta}{dz}, \quad (3.11)$$

where

$$\begin{aligned} \bar{A}_\nu &= -\frac{2m^2\nu^2\alpha(x_+ - x_-) + \omega k(\nu^2 + 3)^2}{4m^2\nu^2\alpha^2 + k^2(\nu^2 + 3)^2}, \\ \bar{B}_\nu &= \frac{4m^2\nu^2\alpha(x_+ - x_-)}{4m^2\nu^2\alpha^2 + k^2(\nu^2 + 3)^2}, \\ \bar{C}_\nu &= \frac{2m\nu\alpha(\nu^2 + 3)(x_+ - x_-)}{4m^2\nu^2\alpha^2 + k^2(\nu^2 + 3)^2}. \end{aligned} \quad (3.12)$$

Using the facts [16]

$$azF(a+1, b+1, c+1, z) = cF(a, b+1, c, z) - cF(a, b, c, z), \quad (3.13)$$

and

$$a(1-z)F(a+1, b, c, z) = (c-b)F(a, b-1, c, z) - (c-a-b)F(a, b, c, z), \quad (3.14)$$

the solution can be explicitly given by

$$\begin{aligned}
\phi_\theta &= a_\nu z^{\alpha_\nu} (1-z)^{\beta_\nu+1} F(a_\nu+1, b_\nu+1, c_\nu, z) , \\
\phi_t &= z^{\alpha_\nu} (1-z)^{\beta_\nu} \left\{ [\bar{A}_\nu + (\alpha_\nu + \beta_\nu - b_\nu) \bar{C}_\nu] (c_\nu - b_\nu - 1) F(a_\nu, b_\nu, c_\nu, z) \right. \\
&\quad + [2\beta_\nu(\bar{A}_\nu + (\alpha_\nu + \beta_\nu - b_\nu) \bar{C}_\nu) + a_\nu(c_\nu - a_\nu - 1)] F(a_\nu, b_\nu+1, c_\nu, z) \\
&\quad \left. + a_\nu(\bar{B}_\nu + \beta_\nu \bar{C}_\nu) F(a_\nu+1, b_\nu+1, c_\nu, z) \right\} .
\end{aligned} \tag{3.15}$$

One can also find the leading asymptotic behaviour ( $z \rightarrow 1$ ) of the this solution

$$\begin{aligned}
\phi_\theta &\simeq z^{\alpha_\nu} (1-z)^{-\beta_\nu} \frac{\Gamma(c_\nu) \Gamma(a_\nu + b_\nu - c_\nu + 2)}{\Gamma(a_\nu) \Gamma(b_\nu + 1)} , \\
\phi_t &\simeq z^{\alpha_\nu} (1-z)^{-\beta_\nu} \left\{ [\bar{A}_\nu + (\alpha_\nu + \beta_\nu - b_\nu) \bar{C}_\nu] (c_\nu - b_\nu - 1) (1-z) \frac{\Gamma(c_\nu) \Gamma(a_\nu + b_\nu - c_\nu)}{\Gamma(a_\nu) \Gamma(b_\nu)} \right. \\
&\quad + [2\beta_\nu(\bar{A}_\nu + (\alpha_\nu + \beta_\nu - b_\nu) \bar{C}_\nu) + a_\nu(c_\nu - a_\nu - 1)] \frac{\Gamma(c_\nu) \Gamma(a_\nu + b_\nu - c_\nu + 1)}{\Gamma(a_\nu) \Gamma(b_\nu + 1)} \\
&\quad \left. + (\bar{B}_\nu + \beta_\nu \bar{C}_\nu) (1-z)^{-1} \frac{\Gamma(c_\nu) \Gamma(a_\nu + b_\nu - c_\nu + 2)}{\Gamma(a_\nu) \Gamma(b_\nu + 1)} \right\} .
\end{aligned} \tag{3.16}$$

By imposing the vanishing Dirichlet boundary condition at infinity, one can find the following relation

$$a_\nu = -n , \quad \text{or} \quad b_\nu + 1 = -n , \tag{3.17}$$

which give the quasinormal modes of vector perturbation

$$\begin{aligned}
k &= -i(2\pi T_L)(n + h_L^\nu) , \\
\omega &= -i(2\pi T_R)(n + h_R^\nu) ,
\end{aligned} \tag{3.18}$$

with the left and right conformal weights of the operator dual to vector field

$$h_L^\nu = h_R^\nu + 1 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{3(1-\nu^2)}{4\nu^2} \frac{k^2}{\alpha^2} + \frac{m^2 - 2m\nu}{\nu^2 + 3}} . \tag{3.19}$$

One can see that these modes are of the same form as the scalar case. But the conformal weight of operator dual to vector perturbation is slightly different from that obtained in [10]. However, one can also conclude that, for the vector perturbation, the quasinormal modes are also predicted by the dual CFT.

#### 4. Quasinormal modes of spinor field perturbation

In this section, we calculate the quasinormal modes of fermionic field perturbation in the background of self-dual warped AdS<sub>3</sub> black hole. We consider the spinor field  $\Psi$  with mass  $m$ , which obeys the covariant Dirac equation

$$\gamma^a e_a^\mu \left( \partial_\mu + \frac{1}{2} \omega_\mu^{ab} \Sigma_{ab} \right) \Psi + m \Psi = 0 , \tag{4.1}$$

where  $\omega_\mu^{ab}$  is the spin connection, which can be given in terms of the tetrad  $e_a^\mu$ ,  $\Sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ , and  $\gamma^0 = i\sigma^2$ ,  $\gamma^1 = \sigma^1$ ,  $\gamma^2 = \sigma^3$ , where the matrices  $\sigma^k$  are the Pauli matrices.

According to the metric of self-dual warped black hole, the tetrad field can be selected to be

$$\begin{aligned} e^0 &= \sqrt{\frac{(x-x_+)(x-x_-)}{\nu^2+3}} d\tau, \\ e^1 &= \frac{1}{\sqrt{\nu^2+3}\sqrt{(x-x_+)(x-x_-)}} dx, \\ e^2 &= \frac{2\nu\alpha}{\nu^2+3} d\theta + \frac{\nu}{\nu^2+3} [(x-x_+) + (x-x_-)] d\tau. \end{aligned} \quad (4.2)$$

By employing the Cartan structure equation  $de^a + \omega^a_b \wedge e^b = 0$ , one can calculate the spin connection directly. The nonvanishing components of the spin connection are listed as follows

$$\begin{aligned} \omega_\tau^{01} &= \frac{3-\nu^2}{2(\nu^2+3)} ((x-x_+) + (x-x_-)), \\ \omega_\tau^{12} &= -\frac{\nu}{\sqrt{\nu^2+3}} \sqrt{(x-x_+)(x-x_-)}, \\ \omega_x^{02} &= -\frac{\nu}{\sqrt{\nu^2+3}} \frac{1}{\sqrt{(x-x_+)(x-x_-)}}, \\ \omega_\theta^{01} &= -\frac{2\nu^2\alpha}{\nu^2+3}. \end{aligned} \quad (4.3)$$

The inverse of the tetrad field is given by

$$\begin{aligned} e_0 &= \sqrt{\frac{\nu^2+3}{(x-x_+)(x-x_-)}} \left( \frac{\partial}{\partial\tau} - \frac{[(x-x_+) + (x-x_-)]}{2\alpha} \frac{\partial}{\partial\theta} \right), \\ e_1 &= \sqrt{\nu^2+3} \sqrt{(x-x_+)(x-x_-)} \frac{\partial}{\partial x}, \\ e_2 &= \frac{\nu^2+3}{2\nu\alpha} \frac{\partial}{\partial\theta}. \end{aligned} \quad (4.4)$$

Assuming that the spinor field takes the form  $\Psi = (\psi_+(x), \psi_-(x))e^{-i\omega\tau+ik\theta}$  and changing the variables to  $z$ , one can finally derive the following equations of motion after some algebra

$$\begin{aligned} z^{\frac{1}{2}}(1-z) \frac{d\psi_+}{dz} &+ \left[ \left( \frac{i\omega}{x_+ - x_-} + \frac{ik}{2\alpha} + \frac{1}{4} \right) z^{-\frac{1}{2}} + \left( -\frac{i\omega}{x_+ - x_-} + \frac{ik}{2\alpha} + \frac{1}{4} \right) z^{\frac{1}{2}} \right] \psi_+ \\ &+ \left( -\frac{ik\sqrt{\nu^2+3}}{2\nu\alpha} - \frac{\nu}{2\sqrt{\nu^2+3}} + \frac{m}{\sqrt{\nu^2+3}} \right) \psi_- = 0, \\ z^{\frac{1}{2}}(1-z) \frac{d\psi_-}{dz} &+ \left[ \left( -\frac{i\omega}{x_+ - x_-} - \frac{ik}{2\alpha} + \frac{1}{4} \right) z^{-\frac{1}{2}} + \left( \frac{i\omega}{x_+ - x_-} - \frac{ik}{2\alpha} + \frac{1}{4} \right) z^{\frac{1}{2}} \right] \psi_- \\ &+ \left( \frac{ik\sqrt{\nu^2+3}}{2\nu\alpha} - \frac{\nu}{2\sqrt{\nu^2+3}} + \frac{m}{\sqrt{\nu^2+3}} \right) \psi_+ = 0. \end{aligned} \quad (4.5)$$



The above equation can also be solved by the hypergeometric function. The solution with the ingoing boundary condition is explicitly given by

$$\begin{aligned}\psi_+ &= c_f z^{\alpha_f + \frac{1}{2}} (1-z)^{\beta_f} F(a_f, b_f, c_f, z), \\ \psi_- &= z^{\alpha_f + \frac{1}{2}} (1-z)^{\beta_f} [c_f F(a_f, b_f, c_f, z) - b_f (1-z) F(a_f + 1, b_f + 1, c_f + 1, z)] \\ &= (c_f - b_f) z^{\alpha_f + \frac{1}{2}} (1-z)^{\beta_f} F(a_f + 1, b_f, c_f + 1, z),\end{aligned}\tag{4.6}$$

where

$$\begin{aligned}\alpha_f &= -i \left( \frac{\omega}{x_+ - x_-} + \frac{k}{2\alpha} \right) - \frac{1}{4}, \\ \beta_f &= \frac{1}{2} - \sqrt{\frac{3(1-\nu^2)}{4\nu^2} \frac{k^2}{\alpha^2} + \frac{m - \frac{\nu}{2}}{\nu^2 + 3}}, \\ \gamma_f &= i \left( \frac{\omega}{x_+ - x_-} - \frac{k}{2\alpha} \right) - \frac{1}{4}, \\ a_f &= \alpha_f + \beta_f + \gamma_f, \\ b_f &= \alpha_f + \beta_f - \gamma_f, \\ c_f &= 2\alpha_f + 1.\end{aligned}\tag{4.7}$$

One can also find the leading asymptotic behaviour ( $z \rightarrow 1$ ) of the this solution

$$\begin{aligned}\psi_+ &\simeq c_f z^{\alpha_f + \frac{1}{2}} (1-z)^{\beta_f} \frac{\Gamma(c_f) \Gamma(c_f - a_f - b_f)}{\Gamma(c_f - a_f) \Gamma(c_f - b_f)}, \\ \psi_- &\simeq z^{\alpha_f + \frac{1}{2}} (1-z)^{\beta_f} \frac{\Gamma(c_f + 1) \Gamma(c_f - a_f - b_f)}{\Gamma(c_f - a_f) \Gamma(c_f - b_f)}.\end{aligned}\tag{4.8}$$

By imposing the vanishing Dirichlet boundary condition at infinity, one can find the following relation

$$c_f - a_f = -n, \quad c_f - b_f = -n,\tag{4.9}$$

which give the quasinormal modes of spinor perturbation

$$\begin{aligned}k &= -i(2\pi T_L)(n + h_L^f), \\ \omega &= -i(2\pi T_R)(n + h_R^f),\end{aligned}\tag{4.10}$$

with the left and right conformal weight of the operator in the dual CFT dual to spinor fields

$$h_L^f = h_R^f - \frac{1}{2} = \sqrt{\frac{3(1-\nu^2)}{4\nu^2} \frac{k^2}{\alpha^2} + \frac{m - \frac{\nu}{2}}{\nu^2 + 3}}.\tag{4.11}$$

Again, for the spinor perturbation, the quasinormal modes are of the same form as the scalar and vector cases. The conformal weight of operator in the dual CFT is also of the same form as that obtained in [10]. It is shown that the quasinormal modes of spinor perturbation are also exactly predicted by the dual CFT.

## 5. Discussion

We have studied the scalar, vector and spinor field perturbations of self-dual warped AdS<sub>3</sub> black hole and obtained the exact expressions of the corresponding quasinormal modes. It is shown that the quasinormal modes of various perturbations are just of the forms predicted by the dual CFT. The results may provide a quantitative test of the warped AdS/CFT correspondence.

At last, let us make some comments on the gravitational perturbations in warped AdS black hole background. TMG is an extension of three dimensional Einstein gravity with propagating degrees of freedom. It is interesting to consider gravitational perturbation and study the stability for black holes which is asymptotic to warped AdS<sub>3</sub>. The differential equation of gravitational perturbation in TMG is generally third order, and is rather complicated to solve. The perturbation equation in the background of warped AdS black hole is given by

$$\delta R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\delta R + 2h_{\mu\nu} + \frac{1}{3\nu}\delta C_{\mu\nu} = 0, \quad (5.1)$$

where

$$\begin{aligned} \delta C_{\mu\nu} = & \epsilon_\mu^{\alpha\beta}\nabla_\alpha \left( \delta R_{\beta\nu} - \frac{1}{4}g_{\beta\nu}\delta R - \frac{1}{4}R h_{\beta\nu} \right) \\ & - \epsilon_\mu^{\alpha\lambda}\delta\Gamma_{\alpha\nu}^\beta \left( R_{\lambda\beta} - \frac{1}{4}g_{\lambda\beta}R \right) \\ & + \left( h_{\mu\gamma}\epsilon^{\gamma\alpha\lambda} - \frac{1}{2}h\epsilon_\mu^{\alpha\lambda} \right) \nabla_\alpha \left( R_{\lambda\nu} - \frac{1}{4}g_{\lambda\nu}R \right), \end{aligned} \quad (5.2)$$

and

$$\begin{aligned} \delta\Gamma_{\alpha\beta}^\lambda = & \frac{1}{2} \left( \nabla_\beta h^\lambda_{\alpha} + \nabla_\alpha h^\lambda_{\beta} - \nabla^\lambda h_{\alpha\beta} \right), \\ \delta R_{\mu\nu} = & \frac{1}{2} \left( \nabla_\lambda \nabla_\mu h^\lambda_{\nu} + \nabla_\lambda \nabla_\nu h^\lambda_{\mu} - \nabla^2 h_{\mu\nu} - \nabla_\mu \nabla_\nu h \right), \\ \delta R = & \nabla^\mu \nabla^\nu h_{\mu\nu} - \nabla^2 h. \end{aligned} \quad (5.3)$$

Generally, it is believed that the equations of motion can be simplified after fixing the gauge conditions. This indeed happens in the case of BTZ black hole [17] and warped AdS<sub>3</sub> vacua [18]. For the case of black holes in warped AdS background, we do not know how to fix the gauge conditions and simplify the equations of motion.

For BTZ black hole [17], which is asymptotic to AdS space, the equations of motion can be split into the second order equation for massless graviton and the first order equation for massive graviton. The quasinormal modes spectrum for gravitational perturbation can be constructed from the chiral highest weight modes by using the  $SL(2, R)_L \times SL(2, R)_R$  symmetry of BTZ background. The extremal case have also been investigated in [19].

More recently, Chen et al in [20] have calculated the quasinormal modes of warped AdS black holes for scalar, vector and tensor perturbations by using the algebraic method developed in [17]. For the scalar and vector perturbations, our results for quasinormal

modes are recovered. For the tensor perturbation, they considered the massive tensor field in warped AdS black holes background satisfying the following first order equation of motion

$$\epsilon_\mu^{\alpha\beta}\nabla_\alpha h_{\beta\nu} + m h_{\mu\nu} = 0, \quad (5.4)$$

and obtained the corresponding quasinormal modes. Their calculations depend on the observation of hidden conformal symmetry of warped AdS black holes in [21, 22].

However, this formalism is hard to be generalized to calculate the gravitational perturbation in warped AdS black hole background because of the  $U(1)_L \times SL(2, R)_R$  symmetry of the geometry of this class and the complexity of perturbation equation. So, for the warped AdS black holes, in order to calculate the quasinormal modes of gravitational perturbations, the more general method should be employed. It is definitely a hard work for future considerations.

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